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## First evidence for triaxial superdeformation in $^{168}\text{Hf}$

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### Abstract

Three superdeformed (SD) bands have been found in  $^{168}\text{Hf}$ . Lifetime measurements revealed a large quadrupole moment,  $Q_t \sim 11.4 eb$ , for the strongest band (TSD1). Theoretical calculations using the Ultimate Cranker code predict high-spin SD minima ( $\epsilon_2 \sim 0.43$ ) with stable triaxial deformations of  $\gamma \sim +20^\circ$  and  $\gamma \sim -15^\circ$ . The measured  $Q_t$  value suggests that band TSD1 corresponds, most likely, to a deformation with a positive  $\gamma$  value. This constitutes the first evidence for triaxial superdeformation in an even proton system. © 2001 Published by Elsevier Science B.V.

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Potential energy surface calculations using different approaches, see, e.g., Refs. [1,2], predict that nuclei with  $Z \sim 72$  and  $N \sim 94$  constitute a new region of exotic shapes coexisting with normal prolate deformation ( $\epsilon_2 \sim 0.23$ ). At high spins these nuclei may assume stable triaxial superdeformed (TSD) shapes characterized by different moments of inertia for each of the principal axes with a deformation para-

meter  $\epsilon_2 \sim 0.4$ . Experimentally, such rotational bands have been reported in  $^{163-165}\text{Lu}$ , with two bands in  $^{163}\text{Lu}$  [3–5], eight in  $^{164}\text{Lu}$  [6], and one in  $^{165}\text{Lu}$  [7]. In  $^{163}\text{Lu}$ , lifetime measurements for transitions below spin  $57/2\hbar$  in the strongest TSD band (now seen from spin  $13/2\hbar$  to  $97/2\hbar$ ), using the recoil distance method and lineshape analysis with the Doppler shift attenuation method (DSAM), revealed a large quadrupole moment of  $Q_t \sim 10.7 eb$  [3]. The association of these bands with high-spin TSD minima was inferred through a comparison of measured spectroscopic in-

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formation with theoretical calculations. The most significant experimental observables include the quadrupole moments, the quasiparticle energies, and the dynamic moments of inertia, which are all influenced by triaxiality. More recent systematic total energy surface calculations [7,8] with the Ultimate Cranker (UC) code [9] predict superdeformed minima ( $\epsilon_2 \sim 0.4$ ) with  $\gamma \sim \pm 20^\circ$  at high spins for nuclei with  $N \sim 92$ –98 and  $Z \sim 72$ . The calculations also show that these TSD minima are expected for all the four combinations of parity  $\pi$  and signature  $\alpha$  in these nuclei. The large deformation is due not only to the shape-driving effect of the intruder  $\pi i_{13/2}$  orbital, but is also the result of a re-arrangement of the core, i.e., more profound shell structures must be involved. The most important shell structure is found in the neutron system with a major shell gap at  $N = 94$ . Appearing at a highly deformed triaxial shape this gap is almost as large as the spherical gap at  $N = 64$ . Thus, the TSD shapes are expected as a general phenomenon for nuclei in this region centered around  $^{166}\text{Hf}$ . Further taking into account the relative energy between the TSD and the normal deformed (ND) yrast structures,  $^{164}\text{Hf}$  was predicted to be the best candidate [8]. However, extensive searches in both  $^{164}\text{Hf}$  and  $^{166}\text{Hf}$  [10] failed so far to identify TSD bands.

In this Letter, we report on the experimental discovery of three TSD bands in  $^{168}\text{Hf}$ , the first evidence for triaxial superdeformation in an even proton system. Lifetime measurements for all transitions in the strongest band, TSD1, using the fractional centroid shift analysis based on the DSAM method, established that the band has a  $Q_t$  value of  $\sim 11.4 e b$ , which corresponds, most likely, to the calculated TSD minimum with  $\epsilon_2 \sim 0.43$  and  $\gamma \sim +20^\circ$ . The weak population of the bands, as compared to the TSD bands in Lu isotopes, suggests several discrepancies between the experimental observations and the theoretical calculations.

Two experiments were performed using the reaction  $^{96}\text{Zr}(^{76}\text{Ge}, 4n)$  to populate the high spin states in  $^{168}\text{Hf}$ . The 310 MeV germanium beam was provided by the ATLAS accelerator at Argonne National Laboratory. Decay  $\gamma$  rays were detected with the Gammasphere array which consisted of 101 Compton-suppressed Ge spectrometers at the time of the experiments. In the first experiment, a self-supporting thin foil ( $0.67 \text{ mg/cm}^2$ ) of  $^{96}\text{Zr}$  was used as a target. A total

of  $2.2 \times 10^9$  events was collected, with a requirement of  $\geq 5$  suppressed Ge detectors in prompt coincidence. As a result of the data analysis, three presumed TSD bands were found. In order to substantiate the nature of the shapes associated with these bands, a second experiment was carried out to measure the transition lifetimes using the DSAM technique. The target consisted of a thin layer ( $0.67 \text{ mg/cm}^2$ ) of  $^{96}\text{Zr}$  backed by  $21 \text{ mg/cm}^2$  evaporated Au, which slowed down and stopped the recoiling nuclei. A total of  $0.97 \times 10^9$  events, with fold  $\geq 4$ , was collected. The heavymet collimators were placed in front of the Ge spectrometers in both experiments. A beam wobbling mechanism developed at ANL was used to deposit the beam particles to the target evenly in an area of  $4 \text{ mm} \times 5 \text{ mm}$ , as compared to a conventional beam spot of 1–2 mm in diameter. This helped with the heat dissipation in the target and made it possible to use a larger than usual beam current of  $\sim 5 \text{ pA}$  to obtain higher data statistics. Otherwise, the beam current would be limited to a much smaller value,  $\sim 1.5 \text{ pA}$  for the self-supporting target.

In the off-line analysis, the coincidence data from the first experiment were sorted into three-dimensional histograms (cubes) and a four-dimensional histogram (hyper-cube) for detailed spectroscopic studies of  $^{168}\text{Hf}$ . These cubes were gated on  $\gamma$ -ray sum-energy and coincidence fold conditions to enhance the 4n reaction channel. The same data set was also used to study  $^{169}\text{Hf}$  [11]. After extensive searches through the coincidence cubes using the Radware software package [12], three presumed TSD bands were found in  $^{168}\text{Hf}$ . These TSD bands are weakly populated: the strongest band, TSD1, has an intensity of  $0.26 \pm 0.10\%$  of the total intensity feeding the ground state of  $^{168}\text{Hf}$ . Bands TSD2 and TSD3 are weaker,  $0.15 \pm 0.06\%$  and  $0.12 \pm 0.05\%$ , respectively. As shown in Fig. 1, the coincidences between each of these bands and the ND yrast transitions in  $^{168}\text{Hf}$  are seen clearly, and firmly establish that these bands do belong to  $^{168}\text{Hf}$ .

These new bands exhibit many of the characteristics shared by known SD bands in different mass regions such as: (i) long sequences of 12, 11, 9 transitions, respectively, were observed in coincidence with each other in each of the three bands, (ii) bands TSD1, –2, and –3 have average energy spacings of about 53, 52, and 55 keV, respectively, which are similar to those in

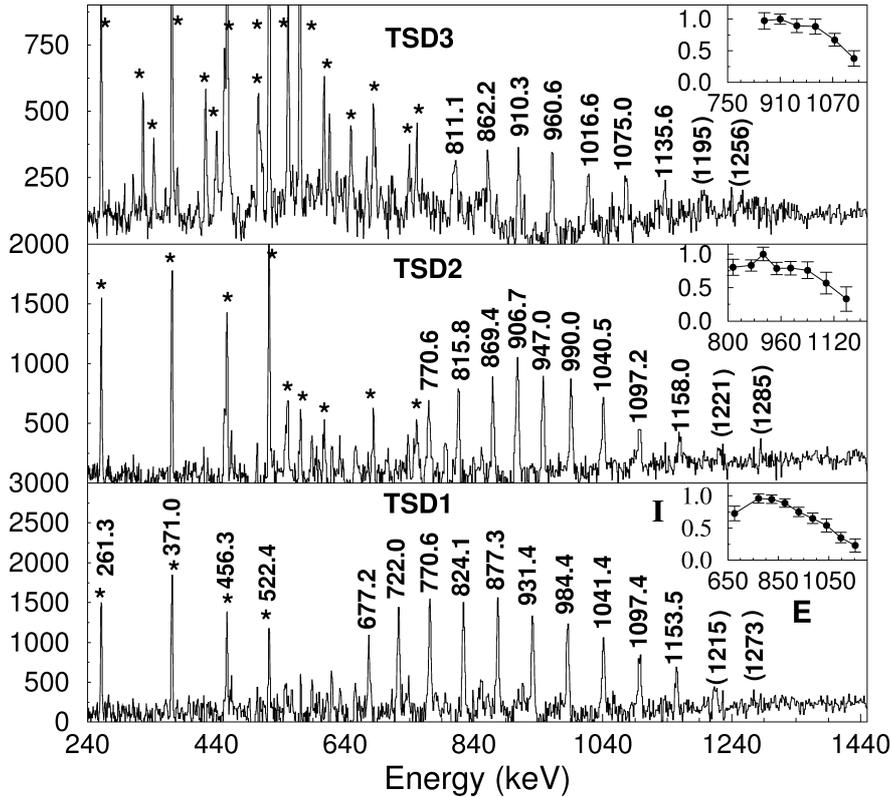


Fig. 1. Efficiency corrected coincidence spectra double gated on combinations of transitions in the TSD bands of  $^{168}\text{Hf}$ . Gamma-ray energies are given for all TSD (marked with energies only) and some ND (marked with the \* symbols) transitions in  $^{168}\text{Hf}$ . The insets show the intensity profile for each TSD band. The detector efficiency curve was normalized to 1.0 at 700 keV.

the TSD bands of  $^{163-165}\text{Lu}$  even though they are less regular (especially in TSD2) than the spacings of many SD bands in the mass 150 and 190 regions, (iii) the bands show the anticipated intensity profile of known SD bands, i.e., the transition intensity increases with decreasing energy until a constant value is reached before a sudden decay occurs over 1–2 transitions at the lowest energies, as illustrated in the insets of Fig. 1.

With the data from the second experiment, it was possible to extract the quadrupole moment only for the strongest band, TSD1. The data were sorted into a number of double gated coincidence spectra, where gates were placed on combinations of transitions in the band TSD1. Spectra corresponding to  $\gamma$  rays detected in each of the 16 angular rings of Gamma-sphere were constructed individually. Linear fits of  $E_\gamma$  versus  $\cos(\theta)$  were then performed using the

first-order Doppler shift expression  $E_\gamma = E_{\gamma 0}[1 + \beta_0 F(\tau) \cos(\theta)]$  to extract the fractional centroid shift values  $F(\tau)$ , where  $\theta$  is the detector angle, and  $\beta_0 = v_0/c = 0.0409$ , is the calculated initial recoil velocity of a residue formed in the center of the target. More details about the procedure can be found in Ref. [13]. The measured  $F(\tau)$  values, as a function of  $\gamma$ -ray energy are presented in Fig. 2. The transition quadrupole moment,  $Q_t$ , for TSD1 was extracted from the experimental  $F(\tau)$  values using the computer code FITFTAU [14]. The code computes the average recoil velocity at which the decay from a particular TSD state occurs with the following assumptions:

(i) All levels within the TSD band have the same  $Q_t$  value, and the transition probability  $T$  (in  $\text{ps}^{-1}$ ) of a band member of spin  $I$  is described within the rotational model by the expression

$$T(I \rightarrow I - 2)$$

$$= 1.22E_\gamma^5(5/16\pi)Q_t^2\langle IK20|(I-2)K\rangle^2$$

assuming  $K = 0$ , where  $E_\gamma$  is the  $\gamma$ -ray energy in MeV.

(ii) The sidefeeding into each TSD state is approximated by a single rotational cascade with the number of transitions in the sidefeeding cascade proportional to the number of transitions in the TSD band above the state of interest. These rotational cascades are assumed to have the same dynamical moment of inertia  $\mathcal{J}^{(2)}$  as the TSD band, and are controlled by a sidefeeding quadrupole moment  $Q_{sf}$  which was assumed to remain the same throughout the entire TSD band.

(iii) A one-step delay at the top of all feeder cascades was parameterized by a single lifetime  $\tau_{sf}$ . The sidefeeding intensities were determined from the  $\gamma$ -ray intensities, extracted from the thin-target measurement, within the TSD band. The stopping powers of the target and the Au backing were calculated using the code TRIM (version 2000) by Ziegler [15]. Finally, a  $\chi^2$  minimization fit using the parameters  $Q_t$ ,  $Q_{sf}$ , and  $\tau_{sf}$  was performed to the measured  $F(\tau)$  values. The results of the fitting are summarized in Fig. 2. The quoted errors in  $Q_t$ ,  $Q_{sf}$ , and  $\tau_{sf}$  include the covariance between the fit parameters. The estimated 10–15% systematic uncertainties associated with the stopping powers are not included. The extracted value of  $Q_t \sim 11.4$  eb, about twice that for the ND states in this nucleus [16], provides a direct measurement of the large deformation associated with the band TSD1.

As no linking transitions from the new bands to the ND states were found, the excitation energies, spins and parities of the TSD states could not be established. Based on the population intensity, band TSD1 is likely to have the lowest, and TSD3 the highest excitation energy above the yrast band. The spins of the bandheads for TSD1,  $-2$  and  $-3$  were estimated to be  $21$ ,  $24$ , and  $28\hbar$ , respectively, to obtain reasonable alignments when compared to the ND structures and to the TSD bands in the Lu isotopes. The estimated values have an uncertainty of  $2$ – $3\hbar$ . Further insight into the nature of these new bands may be obtained by inspecting the behavior of the  $\mathcal{J}^{(2)}$  moments, which are illustrated in Fig. 3. The average  $\mathcal{J}^{(2)}$  values of the new bands are about 16% larger than that of the ND yrast band in this nucleus over the entire region of rotational frequency where these bands are observed.

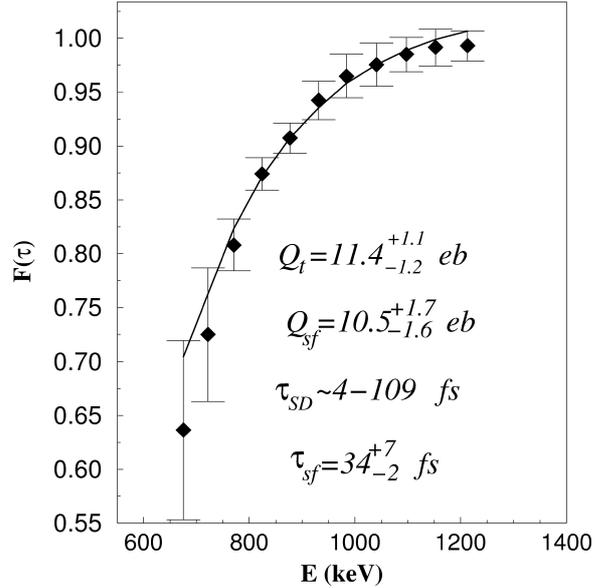


Fig. 2. Measured  $F(\tau)$  values for transitions in band TSD1 of  $^{168}\text{Hf}$ . The solid curve represents the best fit corresponding to the transition quadrupole moment,  $Q_t$ , extracted for this band. Lifetimes associated with the TSD states,  $\tau_{SD}$ , are also presented.

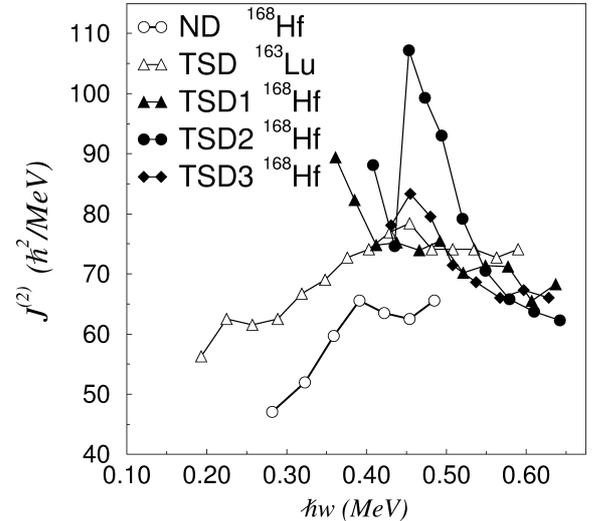


Fig. 3. The dynamic moments of inertia  $\mathcal{J}^{(2)}$ , as a function of the rotational frequency  $\hbar\omega$ , for the newly discovered TSD bands in  $^{168}\text{Hf}$ , the ND yrast band in  $^{168}\text{Hf}$ , and the strongest TSD band in  $^{163}\text{Lu}$ . The high frequency regions for the latter two bands where crossings occur are not shown.

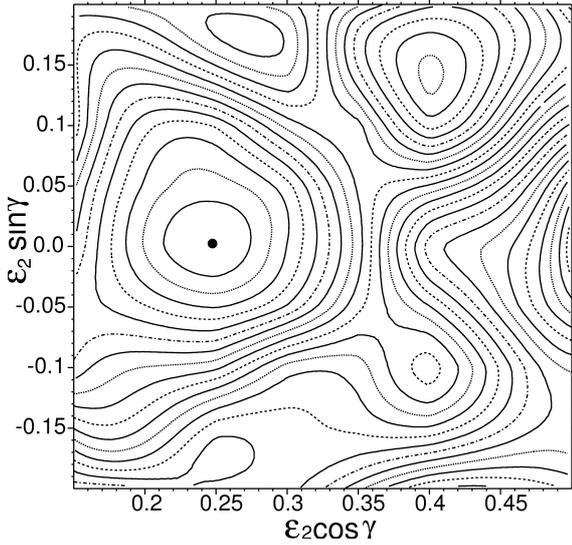


Fig. 4. Calculated (UC) potential energy surface at  $I = 50\hbar$  for the lowest configuration of  $(\pi, \alpha) = (+, 0)$  in  $^{168}\text{Hf}$ . The energy difference between the contour lines is 0.2 MeV.

The  $\mathcal{J}^{(2)}$  values are also about the same as those of the strongest TSD band in  $^{163}\text{Lu}$ . This suggests that all three new bands and the band in  $^{163}\text{Lu}$  have similar, large deformations. At intermediate rotational frequencies, the  $\mathcal{J}^{(2)}$  behavior of bands TSD1 and TSD3 in  $^{168}\text{Hf}$  is very similar to that of the TSD band built on the  $i_{13/2}$  proton orbital in  $^{163}\text{Lu}$ . This may suggest a similar intrinsic configuration for the two bands in  $^{168}\text{Hf}$  since the behavior of  $\mathcal{J}^{(2)}$  is influenced by the number of high- $N$  intruder orbitals occupied, as was first shown by Bengtsson et al. [17]. The  $\mathcal{J}^{(2)}$  behavior of TSD2, on the other hand, is very different. This may indicate a crossing at  $\hbar\omega \sim 0.45$  MeV and, therefore, a different configuration for TSD2. Without detailed information on spins, parities, and excitation energies, however, the configurations associated with these new bands cannot be determined.

Potential energy surfaces have been calculated using the UC code, which provides the tools for a diabatic treatment of crossings between quasiparticle energy levels with a weak or moderate interaction strength. The quasiparticle levels occupied in different configurations are ordered energetically within each combination of  $(\pi, \alpha)$ . Fig. 4 illustrates the calculated surface for the lowest configuration of  $(\pi, \alpha) = (+, 0)$  at spin  $50\hbar$ . The ND minimum is seen at  $(\epsilon, \gamma) =$

$(0.25, 0^\circ)$ , in satisfactory agreement with the measured deformation [16]. There are two TSD minima at  $(0.43, +20^\circ)$  and  $(0.43, -15^\circ)$ . The minimum with  $\gamma \sim +20^\circ$  is about 1.2 MeV lower in energy than the one with  $\gamma \sim -15^\circ$ . Similar minima were seen for other combinations of  $(\pi, \alpha)$ . Nuclei in these two minima, rotating about the smallest ( $\gamma = +20^\circ$ ) and the intermediate axis ( $\gamma = -15^\circ$ ), will have different average quadrupole moments calculated microscopically for the involved quasiparticles to be 10.5  $e b$  and 13.2  $e b$ , respectively. This is significantly larger than the  $\sim 6 e b$  for the ND structure. The calculated quadrupole moments are fairly constant throughout the entire spin range of 20–50  $\hbar$  over which band TSD1 extends. Since the transition quadrupole moment is influenced by the triaxiality associated with the SD bands, the difference between the average quadrupole moments of the bands built on the minima with positive and negative  $\gamma$  values is considered as one of the most significant experimental signatures for triaxiality. The measured quadrupole moment for TSD1,  $\sim 11.4 e b$ , is closer to the value calculated for the minimum with positive  $\gamma$ . However, within the experimental errors, a value  $Q_t \sim 13.2 e b$  corresponding to the minimum with negative  $\gamma$ , cannot be completely ruled out. Considering also the fact that the band associated with a positive  $\gamma$  value has a calculated excitation energy  $\sim 1.2$  MeV lower than the one with negative  $\gamma$ , and will thus become more favorable to observe, we conclude that band TSD1 corresponds, most likely, to the superdeformed minimum with positive  $\gamma$ . Theoretically, there could be another more speculative scenario in which the rotational axis does not coincide with any of the principal axes of the nucleus. This would be an interesting case to which the three-dimensional cranking [18] or the tilted axis cranking model [19] could be applied. However, this scenario is unlikely in  $^{168}\text{Hf}$  as the minimum with positive  $\gamma$  is much deeper than the one with negative  $\gamma$ , and the shapes are not identical.

Calculations of Ref. [8] predict  $^{164,166}\text{Hf}$  to be more favorable to find the TSD shape than  $^{168}\text{Hf}$ . The experimental results seem to suggest a different trend in the evolution with mass of the neutron shell gap. Furthermore, the relative intensities of the strongest TSD bands in  $^{163}\text{Lu}$  and  $^{168}\text{Hf}$  are  $\sim 10\%$  and  $\sim 0.3\%$  as compared to each of their total populations, respectively. Such a large difference seems to imply a greater

role for the proton shell gap than expected from theory. So far, TSD bands have been found in only four isotopes in nuclei with  $N \sim 92\text{--}98$  and  $Z \sim 72$ . A systematic experimental search for TSD bands in this mass region, as well as further theoretical investigations, are clearly needed to clarify these issues. In addition, as indicated previously, the triaxiality associated with the SD bands is currently inferred from comparisons of measured spectroscopic information with theoretical calculations. In general, triaxiality is very difficult to directly determine experimentally. A possible approach would be looking for the so-called wobbling mode [20] which is a direct consequence of the rotational motion of a triaxial body. The wobbling degree of freedom introduces sequences of bands with an increasing number of wobbling quanta, and a characteristic decay pattern between them in competition with the in-band decay [20,21]. Such a study remains a challenge for experimentalists. The band TSD3 in  $^{168}\text{Hf}$  has  $\mathcal{J}^{(2)}$  values similar to those of TSD1. However, it seems very difficult to determine if TSD3 exhibits any of the anticipated characteristics of a wobbling band built on TSD1 because these bands are very weak.

In summary, three SD bands were identified in  $^{168}\text{Hf}$ . Lifetime measurements using the fractional centroid shift method revealed a large transition quadrupole moment of  $Q_t \sim 11.4 e b$  for the strongest band. This provides a direct measure of the large deformation associated with the band. Theoretical calculations predict high-spin SD minima ( $\epsilon_2 \sim 0.43$ ) with stable triaxial deformations of  $\gamma \sim +20^\circ$  and  $\gamma \sim -15^\circ$  in  $^{168}\text{Hf}$ . The measured value of the quadrupole moment suggests that band TSD1 corresponds, most likely, to a positive  $\gamma$  value. This result constitutes the first evidence for triaxial superdeformation in an even proton system of this region.

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